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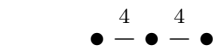
Coxeter Groups

Question 1. Describe the Coxeter complex of $I_2(m)$ for any $m \geq 2$ and for $m = \infty$.

Question 2. Describe the Coxeter complex of $\tilde{B}_2$, whose Coxeter graph is



Strongly suggested tip: play around with https://www.jgibson.id.au/lievis/affine_weyl/.

Question 3. Let $W = S_4$, the symmetric group on $\{1, 2, 3, 4\}$. Then W has the structure of a Coxeter group with $S = \{s_1, s_2, s_3\}$ where s_i denotes the transposition $(i, i + 1)$. Show that $w = s_1 s_2 s_1 s_3 s_2 s_1$ is a reduced expression. Show that for any $s \in S$ we hae $\ell(ws) = \ell(w) - 1$ and find a simple reflection that can be removed from the reduced expression of $\ell(w)$ to give a reduced expression for ws.

Question 4. Let $\{s, t, u\}$ be the simple reflections inside the Coxeter group of type A_3. Show that the subgroup generated by (su) and t is a Coxeter group of type $B_2 = I_2(4)$, with simple reflections $\{su, t\}$, by checking the braid relation.

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Kazhdan–Lusztig polynomials and Kazhdan–Lusztig basis

Question 5. Let W be a dihedral Coxeter group, i.e. $|S| = 2$. Show that all the KL polynomials are trivial, i.e. $h_{x,y}(v) = v^{\ell(y)-\ell(x)}$.

Question 6. Let W the Weyl group of type A_3 with simple reflection s, t, u. Compute the Kazhdan–Lusztig basis element $\underline{H}_{tsut}$.

Question 7. Show that $\underline{H}_x\underline{H}_s = (v + v^{-1})\underline{H}_x$ if $x \in W$ and $s \in S$ with $xs < x$.

Question 8. Let w be the longest element of a finite Coxeter group. Show that for any $x \in w$ we have $h_{x,w}(v) = v^{\ell(w)-\ell(x)}$.

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Combinatorics of KL polynomials

Question 9. Describe the poset structure (for example by drawing the Hasse diagram) of the interval $[t, tsut]$ in type A_3.