Lie Groups
SoSe 2020 — Ubungsblatt 6

Ausgabe 29.06.20, **Abgabe** 14.07.20

Solutions are due on Tuesday 14th July. Please send them by email at

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Aufgabe 6.1: Let $\mathfrak{k}$ be a Lie algebra. Show that we have an isomorphism of Lie algebras $\mathrm{Lie}(\mathrm{Aut}\mathfrak{k}) = \mathrm{Der}(\mathfrak{k})$. where *Aut* is the group of Lie algebra automorphisms of the Lie algebra, and

$$\mathrm{Der}(\mathfrak{k}) = \{d \in \mathfrak{gl}(\mathfrak{k}) \mid d([X,Y]) = [dX,Y] + [X,dY] \text{ for all } X,Y \in \mathfrak{k}\}.$$

Hint: Since $Aut(\mathfrak{k}) \subset GL(\mathfrak{k})$, then $Lie(Aut(\mathfrak{k})) \subset Lie(GL(\mathfrak{k})) = \mathfrak{gl}(\mathfrak{k}))$. Then, as usual, use Satz 1.2.10 from the Skript.

(4 Punkte)

Aufgabe 6.2: Let $\mathfrak{g}$ be a Lie algebra and $I \subset \mathfrak{g}$ be an ideal. Show that the restriction of the Killing form of $\mathfrak{g}$ to I is the Killing form of I.
Hint: Use that if $f : \mathfrak{g} \to \mathfrak{g}$ is such that $\mathrm{Im}(f) \subset I$, then $\mathrm{Tr}(f) = \mathrm{Tr}(f_{|I})$.

(4 Punkte)

Aufgabe 6.3: Show that

$$T = \left\{ \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \mid \theta \in \mathbb{R} \right\}$$

is a maximal torus of $SO_3(\mathbb{R})$.
Hint: Use Exercise 4.4(2)

(4 Punkte)

Aufgabe 6.4: Let K be a compact Lie group. Show that the Lie algebras of the maximal tori in K are precisely the maximal abelian subalgebras of $Lie(K)$.
Hint: Notice that if $\mathfrak{a} \subset Lie(K)$ is an abelian subalgebra, then $\overline{\exp(\mathfrak{a})}$ is a compact connected abelian subgroup of K, hence a torus.

(4 Punkte)