Lie Groups
SoSe 2023 — Ubungsblatt 7

28.06.2023

Aufgabe 7.1: Let G be a finite group.

- Show that the Haar measure on G is
$$\int_G f(g)dg = \frac{1}{|G|}\sum_{g\in G} f(g).$$

- Apply Peter-Weyl theorem on G to deduce
$$|G| = \sum_{V \text{ irred}} (\dim V)^2$$
where the sum runs over all the isomorphism classes of irreducible representations of G.

Aufgabe 7.2: Let G be a compact Lie group and let $\mathfrak{g} = \operatorname{Lie} G$. Show that $\mathfrak{g} = [\mathfrak{g}, \mathfrak{g}] \oplus \mathfrak{z}(g)$.
Hint: Decompose everything in minimal ideal using the adjoint representations. Ideals of dimension one are central.

Aufgabe 7.3: Let K be a compact Lie group. Show that the Lie algebras of the maximal tori in K are precisely the maximal abelian subalgebras of $\operatorname{Lie}(K)$.
Hint: Recall from Aufgabe 3.2 that if $\mathfrak{a} \subset Lie(K)$ is an abelian subalgebra, then $\bar{\exp}(\mathfrak{a})$ is compact and isomorphic to $\mathbb{R}^n/\Gamma$, for some discrete group Γ, hence a torus.

Aufgabe 7.4: Show that $SO_3(\mathbb{R})$ has a maximal abelian group isomorphic to $(\mathbb{Z}/2\mathbb{Z}) \times (\mathbb{Z}/2\mathbb{Z})$. (In particular, this exercises shows that not every maximal abelian subgroup in a compact Lie group is a torus.)