Noncommutative Algebra and Symmetry
WS 2021/22 — Ubungsblatt 5

12.11.2021

Exercise 5.1: Let G be a group and $\rho : G \to GL(V)$ be a representation of G. Show that $\det(\rho(x)) = 1$ for all $x \in (G, G)$.

Exercise 5.2: Let G be a finite group and let χ be the character of a finite dimensional representation of G over $\mathbb{C}$. Show that

$$N := \{g \in G \mid \chi(g) = \chi(1)\}$$

is a normal subgroup of G.
Hint: $\chi(g)$ is the sum of (hom many?) roots of unity.

Exercise 5.3: Let G be a simple group. Show that G does not have an irreducible representation of dimension 2.
Hint: Otherwise $|G|$ must be even (why?) and there exists an element s of order 2. What are the possible eigenvalues of s?

Exercise 5.4: Let Q_8 be the quaternion group with 8 elements $\{1, -1, i, -i, j, -j, k, -k\}$ where the multiplication is defined as follows:

$$(-1)^2 = 1, \quad (-1)x = -x \text{ for all } x$$

$$i^2 = j^2 = k^2 = -1$$

$$ij = k,\ jk = i,\ ki = j$$

$$ji = -k,\ kj = -i,\ ik = -j$$

Compute the character table of Q_8.
(Hint: (-1) is central, and $Q_8/\{\pm 1\} \cong \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$).
Compare it with the character table of D_4. Does the character table determine the group?